MAGNETISM

The magnetic field:

A magnetic Field is a region or space in which:

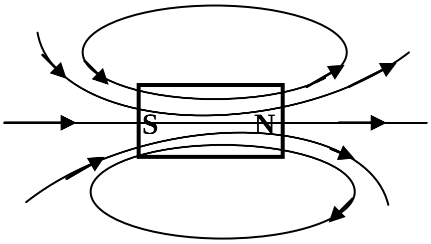
a) a magnetic dipole (magnet) experiences a force.

b) a current carrying conductor experiences a force or a moving charge experiences a force

c) an emf is induced in a moving conductor

Field lines are used to represent the direction and magnitude of the magnetic field. The strength of the magnetic field is proportional to the density of the field lines.

The direction o the magnetic field is represented by the magnetic field lines. The magnetic field lines are taken to pass through the magnet, emerging from the North Pole and returning via the South Pole. The lines are continuous and do not cross each other.



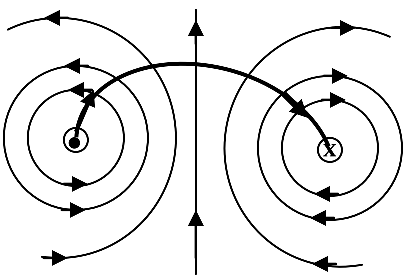
Magnetic fields due to a straight wire carrying current

Maxwell's right hand rule:

This is used to find the direction of the field.

If one grasps the current carrying straight wire in the right hand with the thumb pointing in the direction o current, then the fingers curl pointing in the direction of the magnetic field.

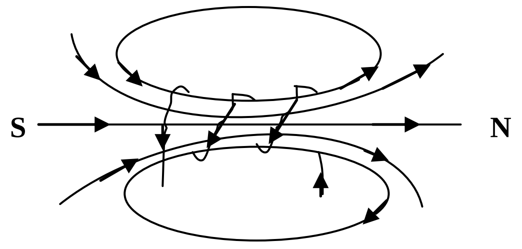
Magnetic field due to a current carrying circular coil



Magnetic fields near the center of the circular coil are uniform hence the magnetic field lines are nearly straight and parallel.

Magnetic fields due to a solenoid

A solenoid can be viewed as consisting o many circular coils, wound very closely to each other.



Use right hand grip to determine direction of field. (fingers show direction of current; thumb direction)

In the middle of the solenoid, the magnetic field lines are parallel to the axis of the solenoid. At the ends of the solenoid, the lines diverge from the axis. The magnetic field due to a current carrying solenoid resembles that of a bar magnet. The polarities of the field are identified by looking at the ends of the solenoid. If current flow is clockwise, the end of the solenoid is South pole, and if anticlockwise then it is North pole.

Magnetic force due to a current carrying conductor in the magnetic field

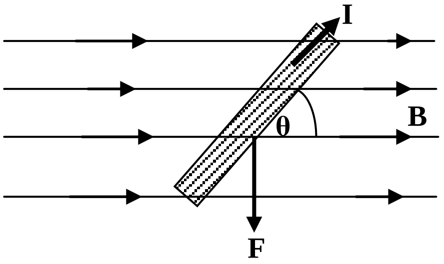
When a current carrying conductor is placed in the magnetic field, a force is exerted on it. Experiments show that the magnitude o the force exerted is proportional to:-

i) the current I in the conductor

ii) the length, *l*, of the conductor

iii) the strength of the magnetic field by a quantity called magnetic flux density, B.

iv) $Sin\ \theta$, Where θ is the angle between the conductor and the magnetic field.



$F \alpha BIlSin\ \theta$

$F = kBIlSin\ \theta$ Where k = constant o rotation.

The unit of B is the *Tesla*(T)

The *Tesla* is the magnetic flux density in a magnetic field when the conductor of length 1m, carrying a current of 1A, at right angles to the magnetic field is 1N.

When B =1T then F = 1N, I = 1A, l = 1m, θ = 90^0

1 = k x1 x 1 x 1 x 1

k = 1

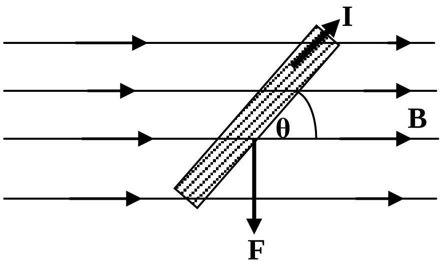
Therefore $F = BIl \sin \theta$

The direction of the force carrying current in the magnetic field is determined using *Fleming's left hand rule.*

If the thumb, the 1^{st} finger and the 2^{nd} finger of the left hand are positioned mutually at right angles, with the 1^{st} finger pointing in the direction of the magnetic field and the 2^{nd} finger pointing in the direction of current, then the thumb points in the direction o the force.

Force of a charge in a magnetic field.

Consider a wire o length l, carrying a current I, in a uniform magnetic field o flux density B.



The current $I = neAV$ where *n* is the number of electrons per unit volume, *A* is the cross-sectional area, *V* is the drift velocity of electrons, and *e* is the electron charge.

The magnetic force on the wire $F = BIl \sin \theta$

but *I =neAV.*

Hence $F = B(neAV)l \sin \theta$

but *nAl = N=* total number of electrons.

$F = BNeV \sin \theta$

but *Ne* = total charge = *q*

$F = BqV \sin \theta$ force on any charged particle.

Force on one electron; $F = BeV \sin \theta$

The force on any charged particle is $F = BqV \sin \theta$.

If the particles velocity is at right angles to $\vec{B}$, $\theta = 90^0$

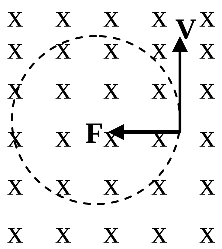
Hence $F = BqV$

$$B = \frac{F}{qV}$$

Hence *magnetic flux density B at a point in a magnetic field, is the force exerted on a charge o 1C, moving with a velocity of 1ms*$^{-1}$*, at right angles to the magnetic field.*

Motion of a charged particle in a uniform magnetic field

Suppose a positively charged particle is projected with velocity V at right angles to a uniform magnetic field o flux density B.



The force $F = BqV$ at right angles to V(using Fleming's left hand rule.)

The rate of change of Kinetic energy (K.E) $\frac{dk}{dt} = \vec{F} \cdot \vec{V} = 0$ since ($\vec{F}$ is perpendicular to $\vec{V}$)

Where k = kinetic energy.

Hence $k = \frac{1}{2} mV^2 =$ constant

It follows that the speed of the charged particle remains constant. Hence the particle moves in a circular path.

For circular motion

$\frac{mV^2}{r} = BqV$. Hence $r = \frac{mV}{Bq}$

Period $T = \frac{2\pi}{\omega} = 2\pi \cdot \frac{r}{V} = \frac{2\pi m}{Bq}$

Frequency, $f = \frac{1}{T} = \frac{Bq}{2\pi m}$

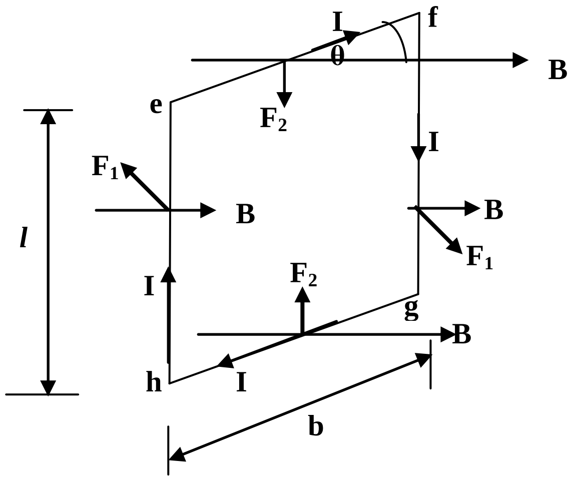
The kinetic energy of the particle is $k = \frac{1}{2} mV^2$

but $V = \frac{Bqr}{m}$

$$k = \frac{1}{2} \frac{B^2 q^2 r^2}{m}$$

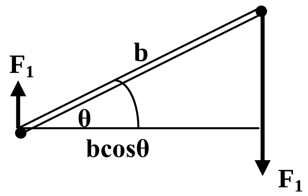
Torque on a current carrying coil in a uniform magnetic field

Consider a rectangular coil carrying a current *I*, in a uniform magnetic field of flux density B. Suppose the plane o the coil makes an angle θ with the magnetic field.



By Fleming's left hand rule, the force on the side *ef* on the coil is $F_2 = BIb\sin\theta$, vertically downwards, whereas the force on *hg* is $F_2 = BIb\sin\theta$, vertically upwards.

The forces F_2, compress the coil and are resisted by the coil's rigidity. On the side *eh*, $F_1 = BIl$, into the plane. The force on *fg* is $F_1 = BIl$, perpendicularly out of the plane o the figure. The forces on *eh* and *fg* are equal and opposite and hence constitute a couple. The moment of this couple, is the torque.



Torque, $\tau = F_1 \cdot b\cos\theta$

but $F_1 = BIl$,

$\tau = BIlb\cos\theta$

but *lb* = A, cross-sectional area of coil.

$\tau = BIA\cos\theta$

For a coil of N turns $\tau = NBIA\cos\theta$.

$\tau = NIBA\cos\theta$.

If α is the angle between the field and the normal to the plane of the coil, then

$\alpha + \theta = 90^0$

$\theta = (90^0 - \alpha)$

$\tau = NBIA\cos(90^0 - \alpha)$

$\tau = NIBA\sin\alpha$

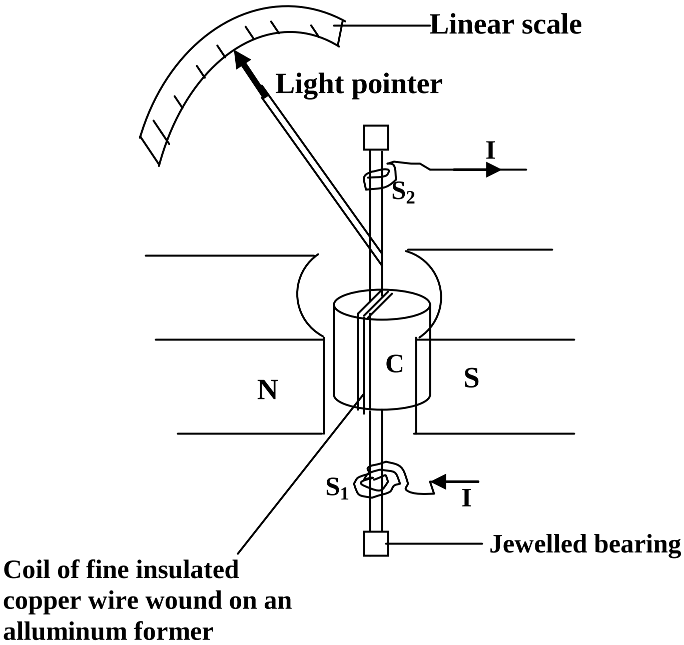
$m = NIA$ is called the magnetic moment of the coil.

When $B = 1, \theta = 0^0$

$\tau = NIA$ Magnetic moment = NIA

Therefore *m*, the magnetic moment is the torque exerted on a coil when it is placed with its plane parallel to a field of magnetic flux density 1T.

MOVING COIL GALVANOMETER



A moving coil galvanometer consists of a rectangular coil of fine insulated copper wire wound on an aluminium frame and pivoted by means of jewel bearing in a radial magnetic field provided by concave soft iron pieces of a powerful permanent magnet. In between the pole pieces lies a cylindrical core C of soft iron.

S_1 and S_2 are springs that provide a resisting torque on the coil and from inlets and outlets for current to be measured.

When the current *I*, to be measured is passed through the coil via spring $S_{1,}$ the torque on coil is $\tau = BANI$; since $\theta = 0$ for radial fields.

The coil will turn until the restoring torque due to twist in the springs is equal to $\tau = BANI$.

The restoring torque is directly proportional to the deflection θ of the coil,

hence restoring torque = $k\theta$.

Where *k* is the torsional constant.

Hence $k\theta = BANI$.

$$\frac{\theta}{I} = \frac{BAN}{k}$$

$\frac{\theta}{I}$ is the current sensitivity.

A radial field in that field in which the plane of the coil in all position remain parallel to the direction of the magnetic field.

Hence $\theta \ \alpha \ I$ therefore a linear scale is obtained.

Factors affecting sensitivity $\left(\frac{\theta}{I}\right)$

A galvanometer is said to be sensitive if it shows a large deflection when a small current passes through it. Hence to obtain a large sensitivity:-

i) N should be large (number of turns)

ii) Area, A should be large

iii) B should be large (strong magnetic field)

iv) k should be small (hence use weak springs which can easily turn)

Note

1: Since large A and N imply a bigger and heavier coil, and yet a large B implies a small gap between the poles of the permanent magnet, a compromise must be sought.

2: The magnetic field is made radial so as to obtain a linear scale.

3: The coil is wound on a metal frame so that eddy currents induced in the coil when it is in motion cause damping, thus making the instrument dead beat.

(Dead beat means that coil comes to rest in the shortest time possible)

If a coil just moves to its final position without oscillating about it, it is said to have a dead beat action.

The aluminium frames help damp oscillations of the coil so that the coil doesn't oscillate.

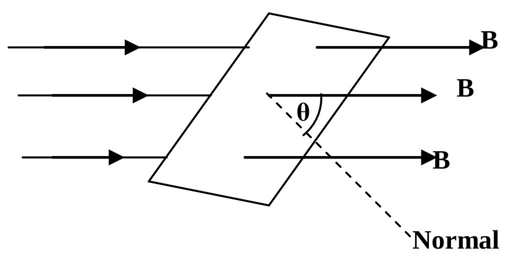
To achieve a linear scale, the magnetic field is made radial by making the poles semi-circular and placing the soft iron cylinder at the center or between the poles.

The soft iron also helps to concentrate the field.

Eddy currents are formed when a conductor is rotated in a uniform field or placed in a changing field.

MAGNETIC FLUX

Consider an area A, the normal of which makes an angle θ with the uniform magnetic field of flux density B.



The component of B along the normal to the area is $B\cos\theta$. The product $\phi = A\cdot(B\cos\theta)$ is called the magnetic flux through the area. Hence magnetic flux is the product of the area and the magnetic flux density along the normal to the area.

If the magnetic field is perpendicular to the area A, then magnetic flux $\phi = AB\cos\theta$,

but $\theta = 0$.   Thus $B = \dfrac{\phi}{A}$

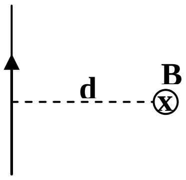
Magnetic flux density is the magnetic flux threading an area of 1 m^2, placed with its plane perpendicular to the magnetic field.

For a coil of N turns, the total flux linking the coil is called the magnetic flux linkage. The unit of magnetic flux is the *Webber (Wb).*

Magnetic flux linkage $\Phi = N\phi = NAB\cos\theta$

Magnetic flux density of current carrying conductors

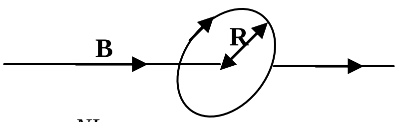
(i) ***Magnetic flux density at a point a distance d from a long straight wire carrying current, I.***



$$B = \frac{\mu_0 I}{2\pi d}$$ where μ_0 is the permeability in free space.

$(\mu_0 = 4\pi\times10^{-7}\,Hm^{-1})$

(ii) ***Magnetic flux density at the center of a circular path of N turns each of turns each of radius R and carrying a current I.***



$$B = \frac{\mu_0 NI}{2R}$$

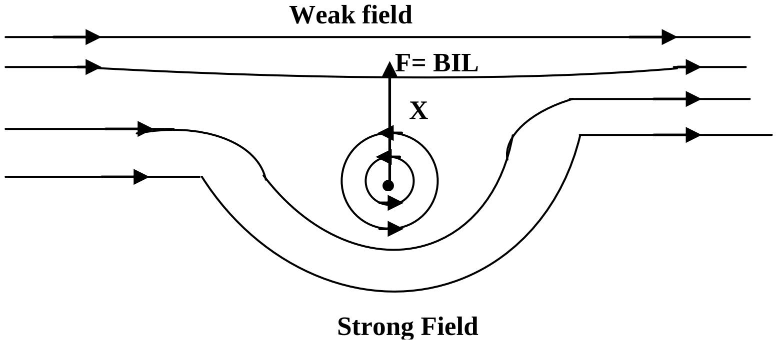
(iii) ***Magnetic flux density along the axis of a long solenoid of n turns per meter, each carrying a current I.***

$$B = \mu_0 nI = \frac{\mu_o NI}{l}$$

Where N is the number of turns and l is the length o the solenoid.

Magnetic force in a current carrying straight line

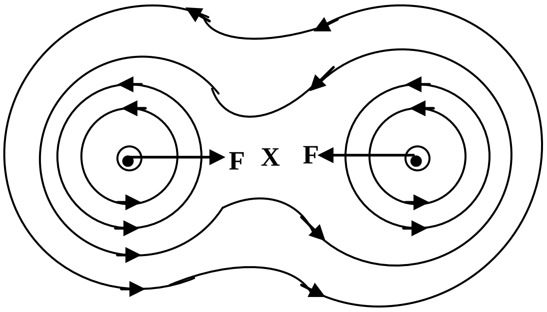
Consider a single straight wire carrying current at right angles to a uniform magnetic field.



X is neutral point

The magnetic force $F = BIl$ on the wire is in that direction which would take the wire from the region of strong field of weak field.

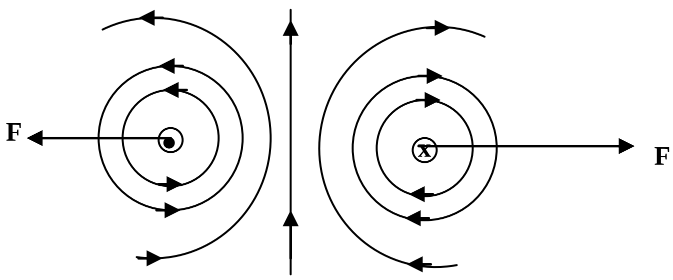
Magnetic field due to two straight wires carrying current in the same direction



X is neutral point

A force on each wire acts from a region of strong field hence straight parallel wires carrying current to the same direction attract i.e. "like currents attract"

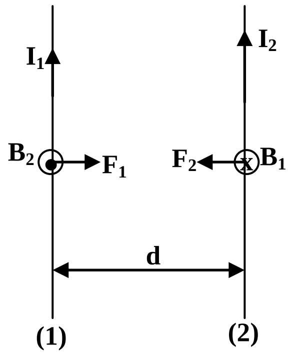
Magnetic field due to straight wires carrying unlike current



"Unlike currents repel"

Magnitude of the force between two wires carrying current.

Consider like currents.



The magnetic flux density B_2 due to current I_2 is given by $B_2 = \frac{\mu_0 I_2}{2\pi d}$.

The force per unit length F_1 on wire 1 due to current I_1 *is* given by $F_1 = B_2 I_1 = \frac{\mu_0 I_1 I_2}{2\pi d}$

The magnetic flux density B_1 due to current I_1 is given by $B_1 = \frac{\mu_0 I_1}{2\pi d}$.

The force per unit length F_2 on wire 2 due to current I_2 *is* given by $F_2 = B_1 I_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$

Hence $F_1 = F_2 = F = \frac{\mu_0 I_1 I_2}{2\pi d}$

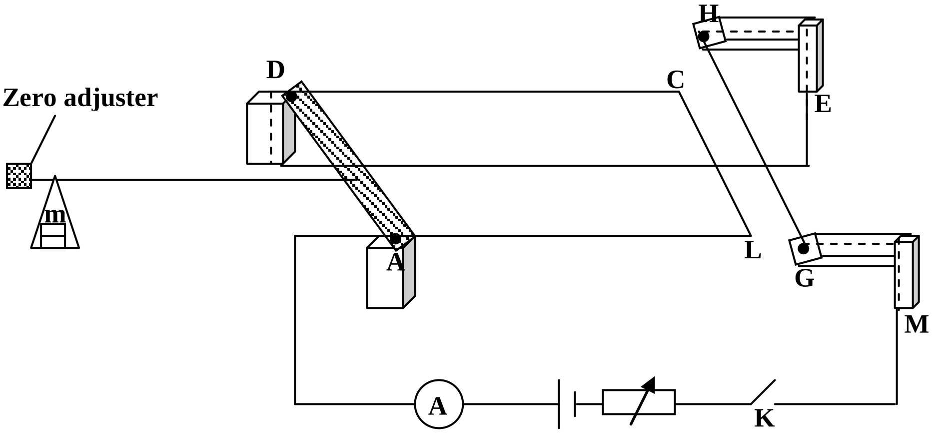
Hence $F_1 = F_2 = \frac{\mu_0 I_1 I_2}{2\pi d}$

If $I_1 = I_2 = 1A$, $d = 1m$,

then $F = \frac{\mu_0}{2\pi} = \frac{4\pi \times 10^{-7}}{2\pi} = 2 \times 10^{-7}\ Nm^{-1}$

Hence the ampere is the steady current which when maintained in each of the two straight and parallel conductors of infinite length and negligible cross section area separated by 1m in a vacuum, produces between the conductors a force of $2 \times 10^{-7}\ Nm^{-1}$.

Absolute measurement of current using the current balance



With no current flowing the zero adjuster is adjusted until plane ALCD is horizontal. The switch is closed, so that current flows through ALCD and EHGM in series. The conductors HG and CL repel, masses m are added to the scale pan to make ALCD horizontal again. The magnetism force between force between HG and CL, $F = \frac{u_0 I^2 l}{2\pi d}$ where *l* is the length of HG and CL and *d* is the suspension of HG and CL when I is 0. The separation *d* is measured using a traveling microscope.

The apparatus is arranged so that the distance of the scale pan from AD is equal to the distance CL from AD.

Hence $mg = \frac{\mu_0 I^2 l}{2\pi d}$.

$$I^2 = \frac{2\pi dmg}{\mu_0 l} = \frac{2\pi dmg}{4\pi x 10^{-7} l}$$

$$I = \sqrt{\frac{dmg}{2x10^{-7} l}}$$

Advantage over ordinary pointer ammeter

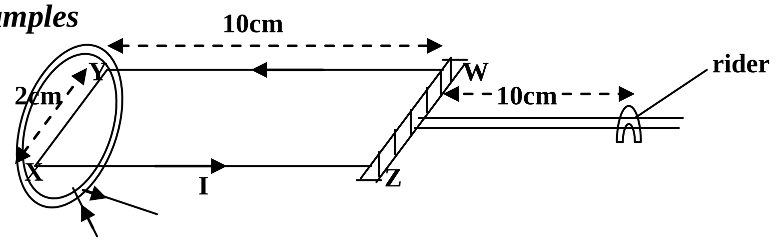
- it gives an accurate method because it involves measurement of fundamental quantities of length and mass.

Disadvantages

- It is not portable
- It doesn't give direct readings and requires a skilled person.

Examples

1.



A rectangular wire WXYZ is balanced horizontally so that the length XY is at the center at a circular coil of 500 turns of means radius 10cm. The a current I is passed through XY and the circular coil, a rider of mass $5x10^{-4}$kg has to be placed at a distance of 9.0cm from WZ to restore balance.

Find the value of I.

Taking moments about WZ.

Fx10 = mgx9

$F = {}^{9}\!/_{10}\, mg$

But $F = BIl$

$B = \frac{u_0 NI}{2R}$

Hence $F = \frac{u_0 NIl}{2R} = \frac{9}{10} mg$.

$I^2 = \frac{0.9 x 5 x 10^{-4} x 9.8 x 2 x 10 \; x 10^{-2}}{4 \pi x 10^{-7} x 500 \;\; x 2 x 10^{-2}}$

$I^2 = 7.019 \; x 10^1$

$I = 8.38 \; A$

2. A current o 1.0A flows in a long solenoid o 100 turns per meter. If the solenoid has a mean diameter of 80cm, find the magnetic flux linkage on one meter length of the solenoid.

$B = \frac{\mu_0 NI}{l} = \frac{4 \text{ X } \pi \text{X} 10^{-1} \text{ X} 1000 \quad \text{X} 1}{1} = 1.275\text{x}10^{-3}$

$\phi = NAB \; \cos \theta = 1000 \; x \left(40 \; x 10^{-2}\right)^2 x \pi x 1.275 \; x 10^{-3} x 1 = 6.317 \; x 10^{-1} Wb$.

3. Two parallel wires each of length 75cm are placed 1.0cm apart. When the same current is passed through the wires, a force of 5.0x 10^{-5}N develops between the wires. Find the magnitude of the current.

$F = \frac{\mu_0 I_1 I_2 l}{2 \pi d}$

$5 \times 10^{-5} = \frac{4 \pi \times 10^{-7} \times I^2 \times 0.75}{2 \pi \times .01}$

$I =$

4. A small circular coil of 10 turns and mean radius of 2.5cm is mounted at the center of a long solenoid. If the current in the solenoid is 2.0A, Calculate:-

(i) The magnetic flux density inside the solenoid.

(ii) The initial torque on the circular coil when a current of1.0A is passed through it.

(i) $B = \mu_0 nI$

$= 4 \pi x 10^{-7} x 750 \; x 20$

$= 1.88\text{x}10^{-3}$T

(ii) $\tau = NIBA \; \cos \theta$

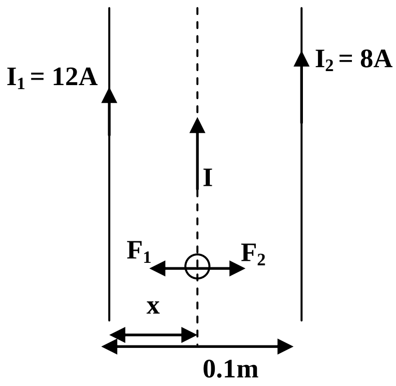
$\tau = NIBA$

$= 1.88 \; x 10^{-3} x \pi x (0.25)^2 x 10 \; x 1$

$=3.7\text{x}10^{-5}$Nm

5. (i) Sketch the magnetic field due to two long parallel conductors carrying respective currents of 12A and 8A in the same direction. (*see notes*)

(ii) If the wires are 10cm apart, find where a third parallel wire also carrying a current must be placed so that the force it experiences is zero.



Let the third wire carry current **I** and be **x m** from wire carrying a current of 12A.

F_1 is force exerted on wire due to current 12A, F_2 is force exerted on wire due to current of 8A.

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$$

$$F_1 = \frac{\mu_0 \times 12\ Il}{2\pi x},\ F_2 = \frac{\mu_0 8 Il}{2\pi(0.1 - x)}$$

If net force is zero , then $F_1 = F_2$

Hence, $\dfrac{\mu_0 \times 12\ Il}{2\pi x} = \dfrac{\mu_0 \times 8 Il}{2\pi(0.1 - x)}$

$$\frac{12}{x} = \frac{8}{(0.1 - x)}$$

$$x = 0.06\ m$$

Exercise

1. The coil of a galvanometer is 0.02m by 0.08m. It consists of 200 turns o wire and is in a magnetic field of 0.2T. The restoring torque constant of the suspension is $1\text{x}10^{-5}$ Nm per degree. Assuming the magnetic field is radial.

(i) What is the maximum current that can be measured by the galvanometer if the scale accommodates a 45^0 deflection?

(ii) What is the smallest current that can be detected if the minimum observed deflection of 0.1^0 ($7.03\text{x}10^{-3}$A, $1.56\text{x}10^{-5}$A)

2 . A moving coil galvanometer has the following characteristics.

- Number of turns = 80
- Area o the coil = $50mm^2$
- Flux density of the radial field = 0.2T.
- Torsional constant of the suspension wire = $5\text{x}10^{-9}$Nm rad^{-1}

- Resistance of the coil = $20\ \Omega$

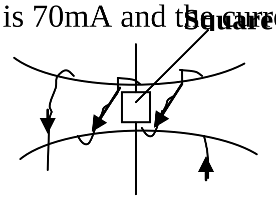
Calculate the angular deflection provided by

(i) a current of 0.01mA (1.6rad)

(ii) a p.d of 0.01mV. (0.08rad)

3. A lat circular coil of 50 turns of mean diameter 40cm is in a fixed vertical plane and has a current of 5Aflowing through it. A small coil 1cm square and having 120 turns is suspended at the center of the circular coil in a vertical plane at an angle of 30^0 to that o the larger coil. Calculate the torque that would act on the small coil when it carries a current o 2ma. ($9.42 x 10^{-9}$)

4. A square coil of side 1.2cm and with 20 turns of fine wire is mounted centrally inside and with its plane parallel to the axis of the long solenoid which has 50 turns per cm. The current in the coil is 70mA and the current in the solenoid is 6.2A.



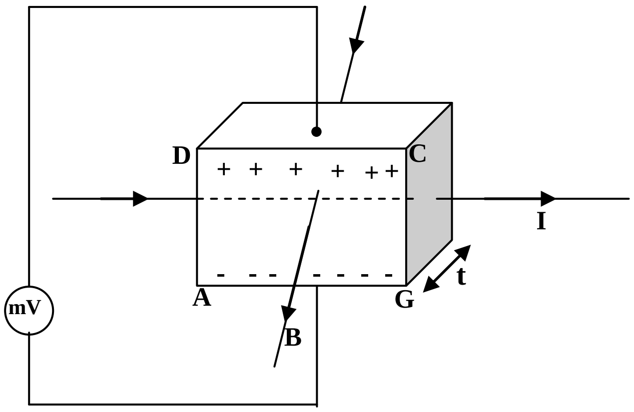
Find

(i) the magnetic flux density inside the coil

(ii) the torque on the square coil

HALL EFFECTS

A current carrying conductor in a magnetic field has a small potential difference across its sides at right angles to the field.



Consider a metal slab carrying current in a magnetic field. The flow of electrons is in opposite direction to a convectional current. If the metal is placed in a magnetic field B, at right angles to the face ACDG of the slab and directed out of paper, a force *BeV* then acts on each electron in the direction from CD to AG. (Fleming's left hand rule)

Therefore electrons accumulate along then side AG of the metal making AG negatively charged and DC positively charged. Hence a p.d or emf which opposes the electron low is set up. This effect is called the *hall effect* (i.e. of producing this emf)

The flow o electrons across ceases when the emf reaches a particular value called *hall voltage.*

The magnitude of Hall Voltage

Force on each electron $= Ee = \frac{V_H e}{d}$ where V_H is the hall voltage.

This force which is directed upwards from AG to DC is equal to the force produced by the magnetic field, when the electrons are in equilibrium. i.e. when the electrons are not deflected.

Hence in equilibrium; $Ee = Bev$

$$E = Bv$$

$$\frac{V_H}{d} = Bv$$

$$\boxed{V_H = Bvd}$$

The drift velocity of the electrons is given by $v = \frac{I}{neA}$

$$V_H = \frac{BId}{neA}$$ But $A = t\text{x}d$.

$$V_H = \frac{BId}{netd} = \frac{BI}{net}.$$

Note: The Hall Effect provides a practical method of measuring magnetic flux density. A given strip (hall probe) is calibrated by measuring the hall voltage V_0 to a given current in a magnetic field of known magnetic flux density B_0. The magnetic flux density B of the unknown magnetic field can then be determined by placing the strip in the magnetic field and measuring the hall voltage V.

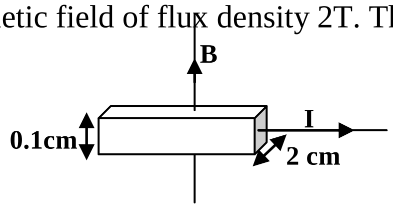
$$V_o = \frac{B_0 I}{net} \qquad V = \frac{BI}{net}$$

$$\boxed{\frac{V}{V_0} = \frac{B}{B_0}}$$

Exercise

1. A metallic strip of width 2.5cm and thickness 0.1cm carries a current of 10amps. When a magnetic is applied normally to the broad side of the strip, a hall voltage of 2mV develops. Find the magnetic flux density if the conduction electron density is $6.0 \times 10^{28} m^{-3}$.

2. A metal strip 2cm wide and 0.1cm tick carries a current of 20A at right angles to a uniform magnetic field of flux density 2T. The hall voltage is 4.27mV.



Calculate

(i) the drift velocity of the electrons in the strip

(ii) the density of charge carriers in the strip.

(i) $V_H = Bvd$

$$v = \frac{V_H}{Bd} = \frac{4.27 \times 10^{-6}}{2 \times 10^{-2} \times 2} = 1.067 \times 10^{-4} ms^{-1}$$

(ii) $V_H = \frac{BI}{net}$

ELECTROMAGNETIC INDUCTION

Electromagnetic induction is the process of getting an emf by changing the number of magnetic field lines associated with the inductor. Electromagnetic induction forms the basis of working of power generation, dynamos, generators etc.

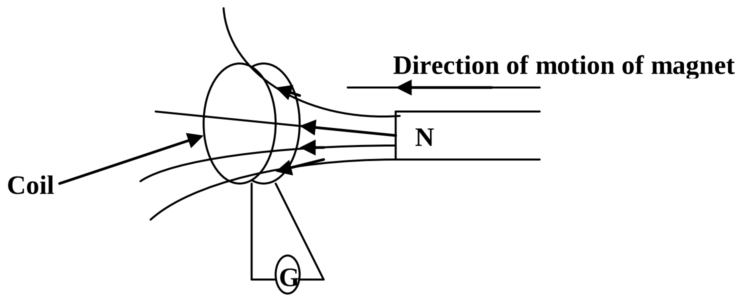
Laws of electromagnetic induction.

(1) Faraday's law:

Whenever the magnetic flux liking a circuit changes, an emf is induced in the circuit. The magnitude of the induced emf is directly proportional to the rate of change of the magnetic flux linking the circuit.

Demonstration of Faraday's experiments

a*) Based on relative movement*



Observations:

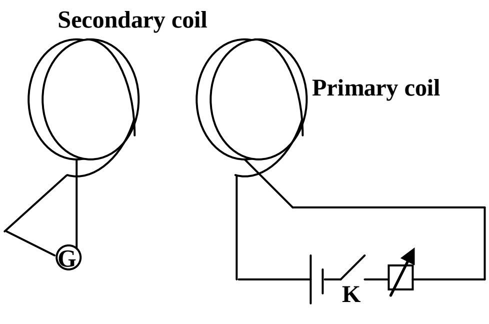
(i) Whenever there is relative motion between a coil and a magnet, the galvanometer shows a certain deflection. This indicates that current is induced in the coil.

(ii) The deflection is temporary. It lasts so long as the relative motion between the coil and the magnet continues.

(iii) The deflection increases with increase in relative motion.

(iv) The direction of the deflection is reversed when either the pole of the magnet is reversed or the direction of motion of the magnet is reversed.

b*) Based on changing magnetic field.*



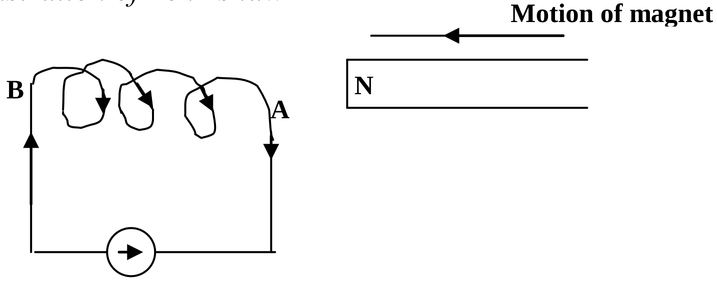
As the switch K is closed, G shows a sudden temporary deflection showing that current is induced in a secondary coil. This is because the current in the primary coil increases from zero to a certain steady value, increasing the magnetic field and hence the number of field lines through the secondary.

When K remains closed, G indicates no deflection, no emf is induced when the magnetic field lines through the secondary remain constant. As K is opened, G shows a sudden temporary deflection in the opposite direction. This is because the decrease in the primary current causes the field lines though the secondary to decrease. Therefore G only deflects when the current in the primary is changing and hence the magnetic flux through the secondary is changing.

Lenz's law:

The direction of the induced emf is in such a way as to oppose the change causing it.

Illustration of Lenz's law



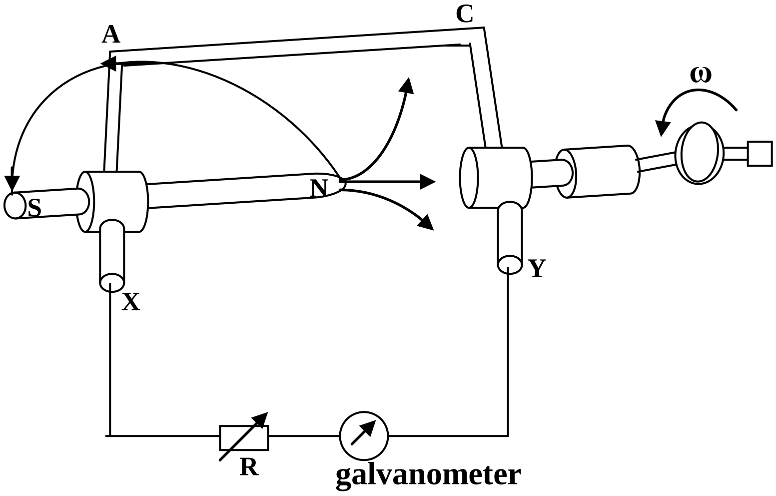
When the North Pole of the magnet is moved towards the coil, the current induced in the coil flows in the direction indicated. And the galvanometer deflects in the clockwise direction. End A of the coil becomes the North Pole. This implies that the fields created by the induced current in the coil opposes the field of the magnet (like poles repel).

When the magnet is moved away from the coil, the galvanometer deflects in the opposite direction, indicating that end A of the coil becomes the South Pole.

Lenz's law and conservation of energy

Lenz's law is an example of conservation o energy. In order not to violate the principle of conservation of energy, the effects of the induced emf must oppose the motion of the magnet, so that the work done by the external agent in moving the magnet is the one converted to electrical energy.

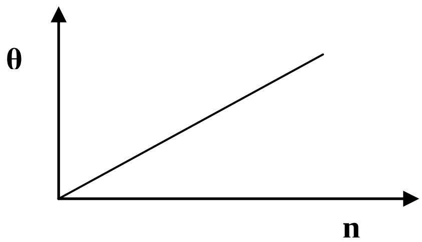
Experiment to show that emf is directly proportional to the rate of change of magnetic flux.



AC is a curved rod which can be rotated by a wheel ω round around the North Pole of a long magnet. Brushing contacts at X and Y connect the rod to the galvanometer and a series resistance, R.

When the wheel is turned, the rod AC cuts across the field, B of the magnet and an emf induced in it. If the wheel is turned steadily, the galvanometer gives a steady reading deflection, θ, showing that a steady current is flowing around a circuit.

Keeping the circuit resistance constant, the rate at which the wheel is turned is varied and the time of revolutions per second is noted and recorded using a stop watch (clock). The number of revolutions per second (n) is obtained each time the wheel is turned. A graph of the deflection θ of the galvanometer against n is plotted and it is a straight line through the origin.



It follows that the induced emf is proportional to the speed of rotation of the rod.

Hence $emf \propto \frac{d\phi}{dt}$, where ϕ is the magnetic flux.

From Faraday's law, induced emf;

$$\varepsilon \alpha -\frac{d\phi}{dt}.$$

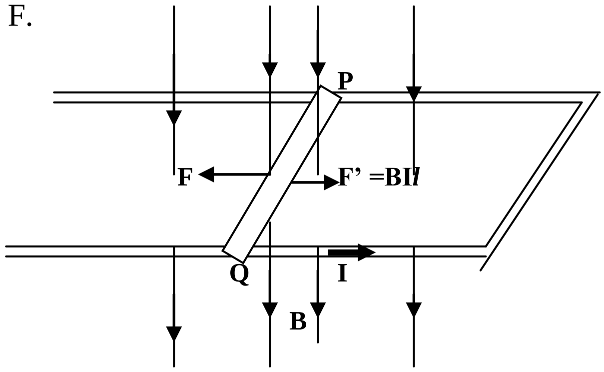
Hence $E = -\frac{d\phi}{dt}$

If φ is the flux threading a conductor in time t, then induced emf.

For a coil of N turns, $E = -\frac{d(N\phi)}{dt}$

Emf induced in a moving conductor

Consider a conducting rod PQ pulled with uniform velocity, *v* of a rectangular frame with a force F.



When PQ is moved to the left with uniform velocity *v*, an induced current flows in the direction shown. Hence PQ becomes a current carrying conductor moving in the magnetic field.

A force $F' = BIl$ is induced on rod PQ, to the right. *l* is the length of PQ.

Since PQ is not accelerating, the two forces are equal i.e. F and F'.

The mechanical power expended is $P = F'v = BIlv$

The electrical power developed $= EI$,where E is the induced emf.

Using the principle of conservation of energy, electrical power = power expended.

Hence $EI = BIlv$

$E = Blv$

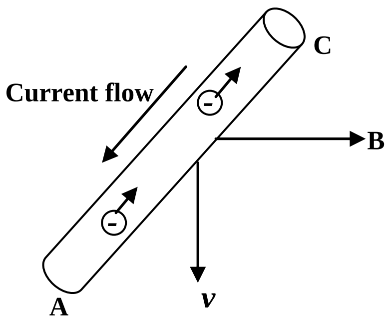
Alternatively, From Faraday's law: $E = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt} = B\frac{dA}{dt}$

But $\frac{dA}{dt} = vl$

Hence $E = -Blv$

$|E| = Blv$

Induced emf and the force on a moving charge.



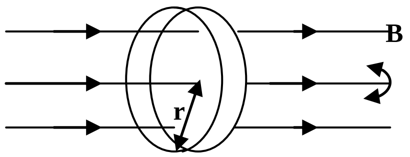
When a wire is moved across the magnetic filed, electrons in it are also moved, likewise across the filed. The force on electrons is at right angles to the plane containing the velocity of the wire, and magnetic flux density, B. hence it tends to drive electrons along the wire. When AC swept across B, the force on the electrons in it acts from A to C. Therefore, if the wire is not connected to a closed circuit, electrons pile up at C. end C hence becomes negative and A positive. End A is at higher potential than C.

Fleming's right hand rule

It is used to determine the direction of the induced current. It states that: "*When the thumb, first finger and second finger of the right hand are held mutually at right angles, with the thumb pointing in the direction of motion, the first finger in the direction of the magnetic filed, then the second finger points in the direction of the induced current".*

Emf induced between the center and the rim of a disc rotating about its axis in a uniform field.

Consider a disc of radius *r*, rotating about its axis at a distance.



Magnetic flux threading the disc is $\phi = BA$ where *A* = area of disc.

The induced emf;

$$E = -\frac{d\phi}{dt} = -\frac{d(BA)}{dt}.$$

$$E = -B.\frac{dA}{dt}$$

In one revolution

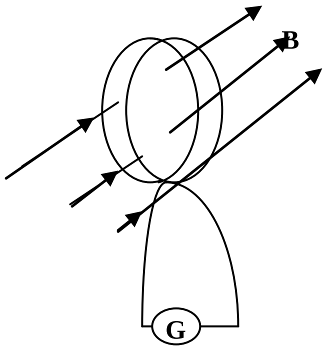
$$\frac{dA}{dt} = \frac{\pi r^2}{T}$$ Where T is period of revolution.

$$E = -\frac{\pi B r^2}{T} = -\pi B r^2 f\ .$$

$$|E| = \pi B r^2 f$$

Relationship between magnetic flux and induced charge.

Consider a coil of N turns each linked by a magnetic flux ϕ_1. Suppose the magnetic flux changes to ϕ_2.



When the magnetic flux ϕ linked with the circuit changes, an emf E is induced in the circuit.

$$E = -N\frac{d\phi}{dt}$$

The induced current $I = \frac{E}{R}$,Where R is the resistance o the coil.

Hence $IR = -N\frac{d\phi}{dt}$

$$I = -\frac{N}{R}\cdot\frac{d\phi}{dt}$$

But $I = \frac{dQ}{dt}$ Where Q is the induced charge.

$$\frac{dQ}{dt} = -\frac{N}{R}\cdot\frac{d\phi}{dt}$$

$$dQ = -\frac{N}{R}d\phi\ .$$

The amount of charge which passes through the coil when the magnetic flux changes from ϕ_1 to ϕ_2 is

$$Q = -\frac{N}{R}\int_{\phi_1}^{\phi_2} d\phi = -\frac{N}{R}(\phi_2 - \phi_1)$$

$$Q = \frac{N(\phi_1 - \phi_2)}{R}$$

$$Q = \frac{\Phi_1 - \Phi_2}{R}$$

Where $\Phi = N\phi$

Induced charge = $\dfrac{\textit{change in flux linkage}}{\textit{Resistance}}$

BALLISTIC GALVANOMETER

The ballistic galvanometer consists of a rectangular coil of fine copper wire wound on a heavy insulating former and having a fine suspension. The coil is made heavy and the suspension fine in order to obtain a large period of oscillation.

The former is made of insulating material to minimize damping which arises due to eddy currents. The coil is suspended in a radial magnetic field between concave magnetic pole pieces attached to a strong permanent magnet.

Mode of operation

When a charge Q flows through the galvanometer, the mass of its coil makes it swing slowly so that the charge finishes circulating while the coil is just beginning to turn. The coil continues to turn and as it turns, it twists the suspension. The coil stops turning when its kinetic energy, which it gained by the forces set up by the current, has been converted into potential energy of a suspension.

Theory shows that if dumping is negligible, the first deflection or "throw" of the galvanometer is directly proportional to the quantity of charge Q , which passes through its coil as it begins to move.

$$Q \ \alpha \ \theta$$

Where θ is the first deflection of the galvanometer.

$$Q = c\theta \text{ ....(i)}$$

Where *c* is the charge sensitivity.

Or $\theta = \alpha Q$ ....(ii)

From (i) and (ii)

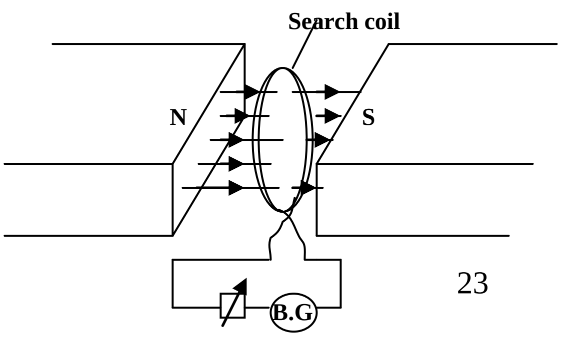
$$\alpha = \frac{1}{c}$$

Unit of c = C rad^{-1} or C div^{-1}

Unit of α = rad C^{-1} or div C^{-1}

Uses of a ballistic galvanometer.

Measurement of magnetic flux density using a search coil and a ballistic galvanometer



A search coil o known geometry i.e. known number of turns N, and known cross-sectional area A, is positioned such that its pane is perpendicular to the magnetic field. It is then pulled out quickly vertically downwards out of the field. The deflection θ of the B.G is noted.

The initial flux linkage of the coil $\phi_1 = NBA$ where B = magnetic flux density of the field.

The final magnetic flux linkage $\Phi_2 = 0$

Hence induced charge $Q = \dfrac{\Phi_1 - \Phi_2}{R}$

Where R is the resistance of the search coil.

$$Q = \frac{NAB - 0}{R}$$

Using the theory of Ballistic Galvanometer, $Q = c\theta$

$c\theta = \dfrac{NAB}{R}$. Hence

$$B = \frac{c\theta R}{NA}$$

The constant c can be determined by discharging a capacitor of capacitance C_0 initially charged to a p.d V_0 through the ballistic galvanometer and noting the first deflection θ_0 . From theory, when it is discharged

$$Q_0 = C_0 V_0 = c\theta_0$$

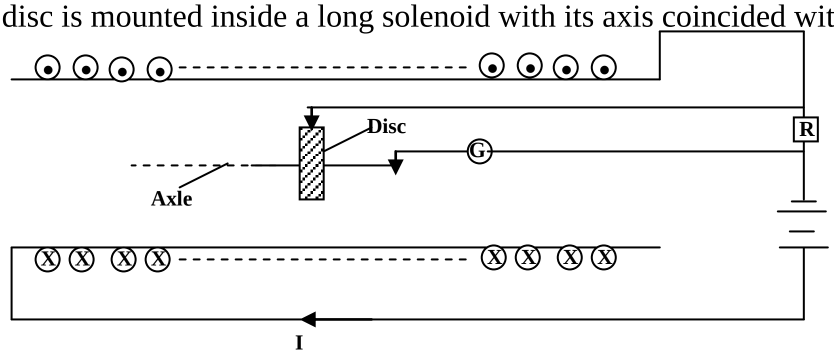
$$c\theta_0 = C_0 V_0$$

Hence $c = \dfrac{C_0 V_0}{\theta_0}$

$$B = \frac{C_0 V_0 \theta R}{\theta_0 NA}$$

Absolute determination of resistance

A metal disc is mounted inside a long solenoid with its axis coincided with that of the solenoid.



The disc is rotated at such a speed that the galvanometer reads 0. When this happens, the induced emf between the rim and the axle of the disc is balanced by the p.d across R.

But

$$E = \pi Br^2 f \text{ , but } f = \frac{\omega}{2\pi}$$

$$= \pi Br^2 \cdot \frac{\omega}{2\pi}$$

$$E = \frac{Br^2\omega}{2}$$

Where r is the radius of the disc, $B = \mu_0 nI$, ω is the angular velocity the disc.

Potential difference across R , $V = IR$.

Hence when current is zero,

$$IR = \frac{Br^2\omega}{2}$$

$$R = \frac{Br^2\omega}{2I}$$

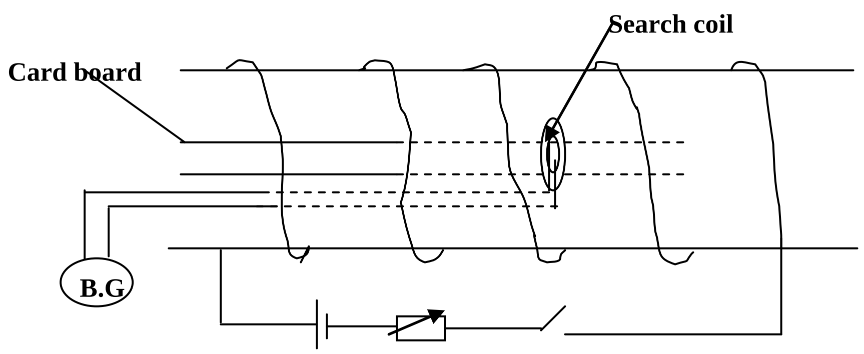
substitute $B = \mu_0 nI$,

$$= \frac{\mu_0 nIr^2 w}{2I}$$

$$\boxed{R = \frac{\mu_0 nr^2 w}{2}}$$

Measurement of magnetic flux density along the axis of the solenoid.

The magnetic flux density B along the axis of a solenoid carrying a current can be measured by means of a search coil connected to a ballistic galvanometer. The search coil is wrapped around a long cardboard which just slides inside the solenoid so that its center is at a number of points spaced about the axis of the solenoid.



At each of the points along the axis the current is switched off and the “throw” θ on the galvanometer is noted.

Induced charge $Q = \frac{N\phi_1 - N\phi_2}{R} = \frac{NAB - 0}{R}$

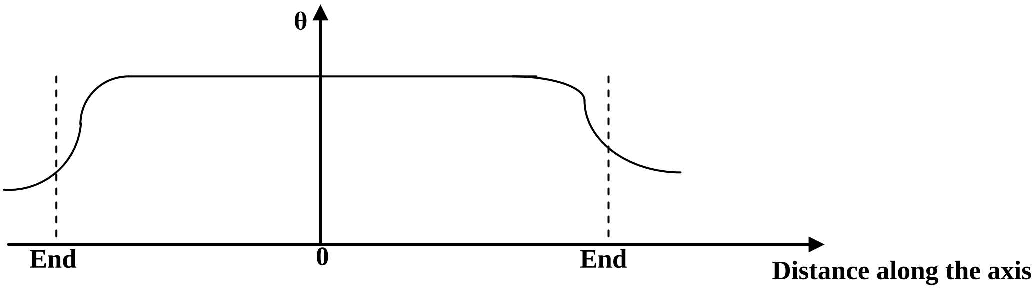
But $Q = c\theta$

$$c\theta = \frac{NAB}{R}$$

$$B = \frac{Rc\theta}{NA}$$

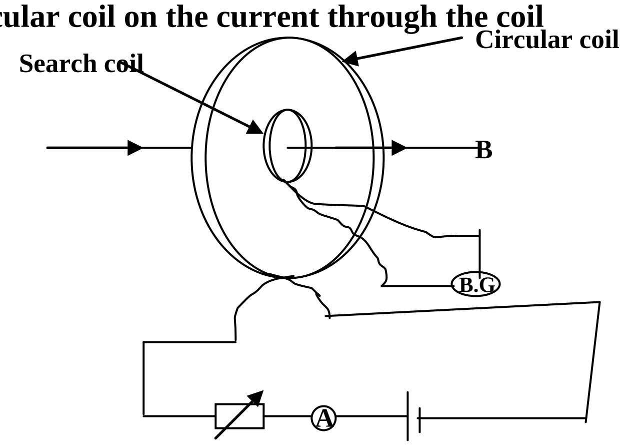
Hence $B \,\alpha\, \theta$

For qualitative investigation, a graph of θ against distance along the axis plotted and this represents the variation of B with the distance.



The flux density B at the end is half that inside the solenoid.

Experiment to investigate the dependence of magnetic flux density *B* at the center of a circular coil on the current through the coil



A small coil called search coil of known number of turns, n, and area, a, is placed at the center of the circular coil and co-planar with it a long current I is passed through the coil, switch s, is opened and the ‘throw’ θ of the galvanometer noted.

Induced charge $Q = \frac{\Phi_i - \Phi_f}{R}$ Where $\Phi_i = nBa$

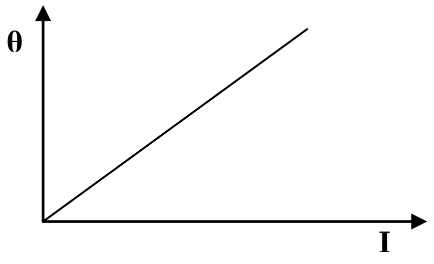
$\Phi_f = 0$

Hence $Q = \dfrac{nBa - 0}{R} = c\theta$

$B = \dfrac{Rc\theta}{na}$

Hence $B \, \alpha \, \theta$

The experiment is repeated for different values of current I and the corresponding 'throw', θ, of the ballistic galvanometer measured. Graph of θ against I is linear showing that the flux density at the center of the coil is directly proportional to the current through it.



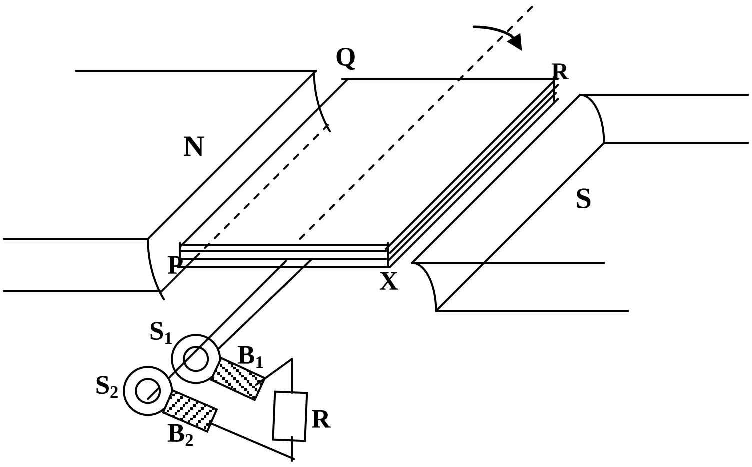
GENERATORS

A generator transforms mechanical energy into electrical energy.

A simple AC generator.

Structure

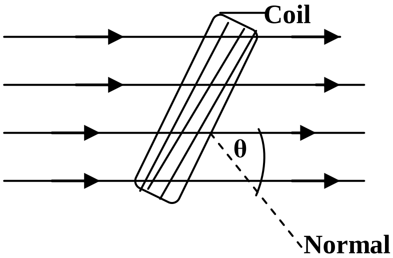
The simple a.c generator consists of a rectangular coil , PQRX, mounted between, N, S- pole pieces of a strong magnet and freely to rotate with uniform angular velocity. The ends of the coil are connected to copper slip rings S_1 and S_2, which press against carbon brushes B_1 and B_2.



Mode of operation

When the coil rotates in the magnetic field, the magnetic flux density linked with it changes and an emf is induced in the coil. The induced emf is led away by means of the slip rings S_1 and S_2.

Consider a coil of N-turns, Area A, whose normal makes an angle θ with a uniform magnetic field B. If $\theta = 0$ when time $t = 0$, then the angular velocity $\omega = \frac{\theta}{t}$. $(\theta = \omega t)$



The magnetic flux linking the coil $\phi = NBA \cos \theta$.

The induced emf in the coil $E = -\frac{d\phi}{dt} = -\frac{d}{dt}(NAB \cos \theta)$

$$E = -NAB \frac{d(\cos \omega t)}{dt} = NA\omega B \sin \omega t = NAB \omega \sin \theta$$

Hence $E = E_0 \sin \omega t = E_o \sin \theta$

Where $E_0 = NAB\ \omega$ = amplitude or peak voltage.

$E_0 = NBA\ \omega$

Factors affecting the emf induced in a rotating coil.

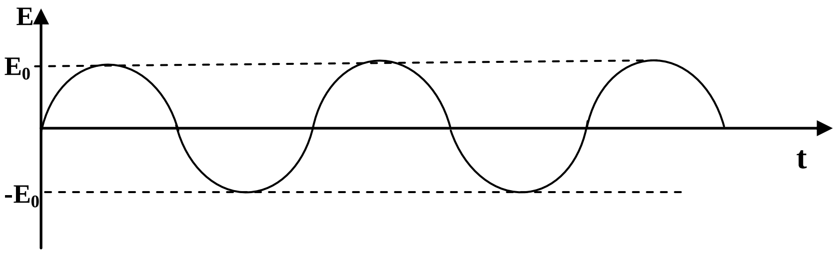
(i) Number of turns on the coil.

(ii) Area of the coil

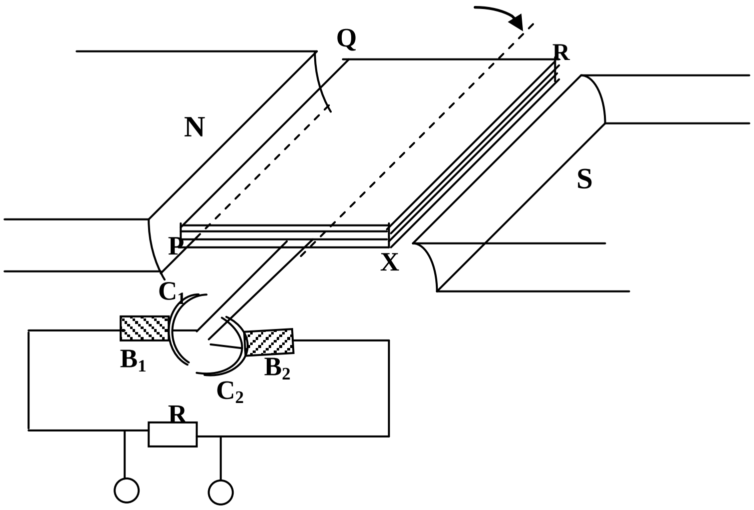
(iii) Magnetic flux density

(iv) Position of the coil

A graph of induced emf E against time



D.C generator

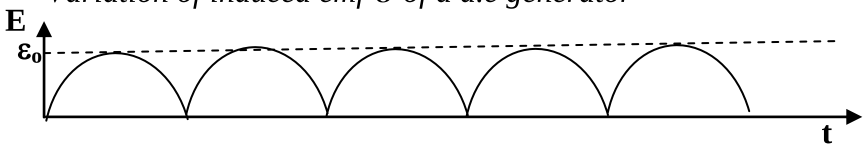


The d.c generator is a device used for producing direct current energy from mechanical energy. The essential parts of a d.c generator are similar to those of an a.c generator except that the slip rings are replaced by a split ring commutator (C_1 and C_2)

Mode of operation

After half the rotation, the slip ring changes contact. C_1 goes into contact with $B_2$and C_2 goes into contact with B_1. Hence the direction of the induced emf doesn't change in the external circuit during one complete revolution of the amateur coil. The output o the generator is unidirectional.

Variation of induced emf ε of a d.c generator



Examples

A flat circular coil with 200 turns each of radius 40cm is rotated at a uniform rate of 480 revolutions per minute about its diameter at right angles to a uniform magnetic field of $5\text{x}10^{-4}$T. calculate

(i) the peak value of emf induced in the coil.

(ii) the instantaneous value of induced emf when the plane of the coil makes an angle of 20° with the field direction.

Solution

$$f = \frac{480}{60} = 8\,Hz$$

i) $E_0 = NAB \times 2\pi f = 2000 \times \pi \times (0.4)^2 \times 5 \times 10^{-4} \times 2 \times \pi \times 8 = 25.3V$

(ii) $E = E_0 \sin \omega t = E_o \sin \theta = 25.3 \times \sin 70 = 23.7V$

Questions:

1. A metal disc of diameter 20cm rotates at constant speed of 60*rev min*$^{-1}$ about an axis through its center and perpendicular to a uniform magnetic field of density $5\text{x}10^{-3}$T established parallel to the axis of rotation. Calculate the emf developed between the axis and the rim of the disc. [1.57mV]
2. A search coil of 20 turns each of area 4cm^2 is connected in series with a ballistic galvanometer and the resistance which makes the total resistance o the circuit 2000Ω. When the search coil is suddenly withdrawn from between the poles of the magnet the galvanometer gives a throw of 20 scale divisions. When a capacitor of capacitance $1\mu F$ changed to a p.d of 2V is discharged through the galvanometer, a throw of 10 divisions is obtained. What is the magnetic flux density between the magnetic poles? [1T]

3. a) A horizontal straight wire 5cm long has a mass of 1.2gm^{-1} is placed perpendicular to uniform horizontal magnetic field of flux of 0.6T. The resistance of the wire is 3.8Ω in the magnetic field.

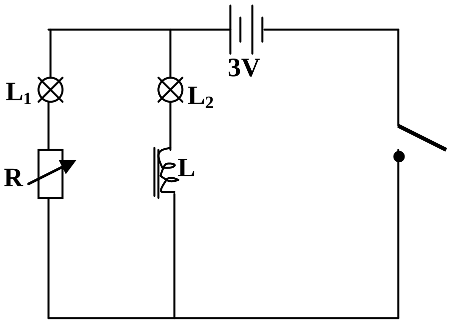
(i) Find the potential difference that must be applied between the ends of the wire to make it selfsupporting.

(ii) Illustrate your answer with a diagram showing the direction of the field and the direction in which the current would have to flow in the wire.

Self Induction

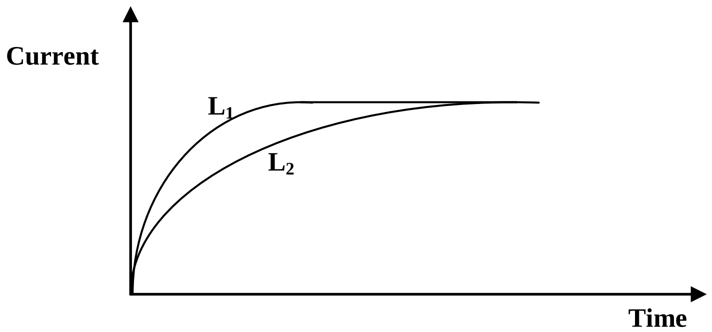
The flux due to the current in the coil links that coil and if the current changes, the resulting flux change induces an emf in the coil itself. This effect is called self induction. The coil is said to have self inductance, (L) and the coil is said to be an inductor. The induced emf tends to oppose the growth of current in the coil.

Demonstration of self induction



L and R have the same resistance. L_1 and L_2 are identical lamps. When the current is switched on, lamp L_2 lights up a second or two later after lamp L_1 has lit. This is because the induced emf in L opposes the current flowing through lamp L_2. Therefore the growth of current in L_2 to its steady value is delayed.

Variation of current with time through L_1 and L_2



If the 3V d.c is replaced by a 3V a.c, the lamp in series with L does not light because the induced emf, due to self induction in L, opposes the applied p.d continuously.

Question: Why L_2 light when the iron core is removed in the coil?

Self inductance, L, of a coil is the ratio of the induced emf to the rate at which the current in the coil changes, hence $L = -\dfrac{\varepsilon}{\dfrac{dI}{dt}}$

Hence induced emf, $\varepsilon = -L\dfrac{dI}{dt}$

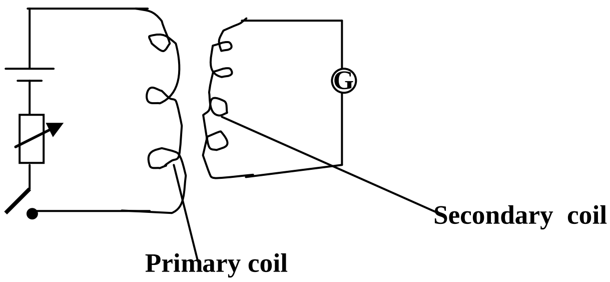
Unit of self inductance is the Henry (H)

Assignment: Prove that the energy stored in an inductor is given by $E = \frac{1}{2} L I_o^2$

Mutual Inductance

If two coils i.e. the primary and the secondary coils, are near each other, and the current in the primary coil is changed, an emf is induced in the secondary coil. This process is called *mutual induction.*

Demonstration of mutual induction



When switch is closed, the galvanometer, G, deflects momentarily, indicating flow of current in secondary coil. This is because the current flowing through the primary coil induces a magnetic field which links up with the secondary coil. When this magnetic field changes, an emf is induced in the secondary coil.

When the switch is opened, the galvanometer deflects momentarily in the opposite direction. This because the induced flux reduces to zero and hence an emf is induced in a direction according to Lenz's law.

Mutual inductance, M between two coils is the ratio of the induced emf in the secondary coil to the rate of change of current in the primary coil.

$M = -\frac{\varepsilon_s}{\frac{dI_p}{dt}}$, ε_s is induced emf in the secondary coil, $\frac{dI_p}{dt}$ is rate of current change in primary coil.

Alternatively, $\varepsilon_s = -\frac{d\Phi_s}{dt}$, where Φ_s is magnetic flux linkage in the secondary coil.

$$\varepsilon_s = -M \frac{dI_s}{dt}$$

hence $M \frac{dI_s}{dt} = \frac{d\Phi_s}{dt}$

$$M = \frac{d\Phi_s}{dI_s} = \frac{\Phi_s}{I_s}$$

hence mutual inductance is the magnetic flux linkage in the secondary coil when the primary current is 1A.

Unit of mutual inductance is the Henry(H).

Mutual inductance of a solenoid and a coil.

Consider a short secondary coil of N_s turns , area A wound round the middle of a long primary solenoid of n turns per metre.

The flux density along the common axis $B = \mu_o n I_p$

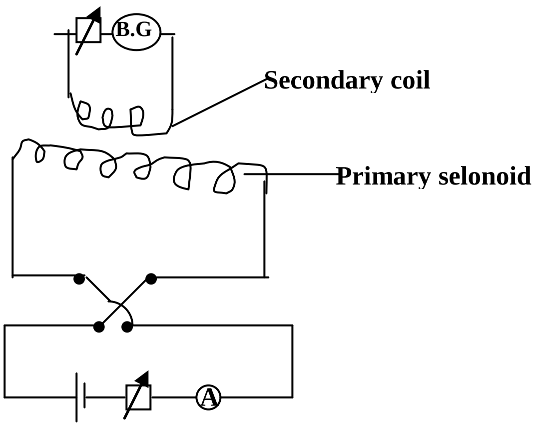
Flux linking the secondary coil, $\Phi_s = N_s BA = N_s \mu_o n I_p A$

$$M = \frac{\Phi_s}{I_s} = N_s \mu_o n A$$

$$M = N_s \mu_o n A$$

Calibration of a Ballistic galvanometer using a standard mutual inductance

A known mutual inductance can be used to calibrate a ballistic galvanometer so that its sensitivity C can be known.



The Ballistic galvanometer is connected to a secondary with a mutual inductance and its deflection θ is noted when a known current I_p as measured by the ammeter is reversed in a primary coil or switched off.

Case I [when current is switched off]

$$\Phi_i = MI_p \text{ , } \Phi_f = 0$$

hence induced charge, $Q = \dfrac{\Phi_i - \Phi_f}{R} = \dfrac{MI_p}{R}$

But from the theory of Ballistic galvanometer, $Q = c\theta$

Hence $c\theta = \dfrac{MI_p}{R}$

Hence $c = \dfrac{MI_p}{R\theta}$

Case II[when I_p is reversed]

$$\Phi_i = MI_p \text{ , } \Phi_f = -MI_p$$

If the deflection of the B.G is θ_1, then induced charge $Q = \frac{\Phi_i - \Phi_f}{R} = \frac{MI_p - -MI_p}{R} = \frac{2MI_p}{R}$

Hence $c\theta_1 = \frac{2MI_p}{R}$

$$c = \frac{2MI_p}{R\theta_1}$$

Examples

1. For calibration purposes, a B.G is connected to the secondary of mutual inductance $1.2\text{x}10^{-5}$H. The total resistance of the secondary circuit being 1000Ω. When a current of 2A is reversed in the primary, the B.G deflects through 30 divisions. What is the sensitivity in Cdiv^{-1}.

$$c = \frac{2MI_p}{R\theta_1} = \frac{2 \times 1.2 \times 10^{-5} \times 2}{1000 \times 30} = 1.6 \times 10^{-9}\,Cdiv^{-1}$$

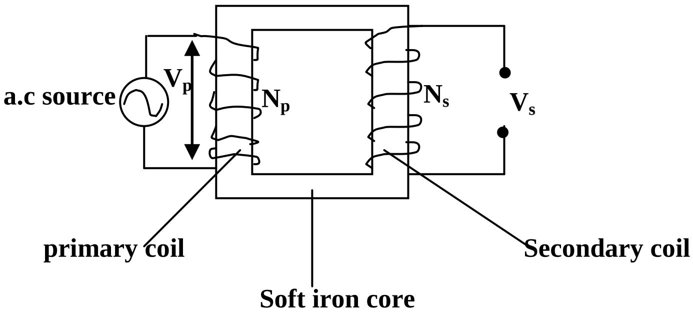
2. A solenoid 50cm long and of mean diameter 3cm is uniformly wound with 2000 turns. A secondary coil of 800 turns is wound closely round the middle of the solenoid and is connected to a B.G. the total resistance of the secondary circuit is 400Ω. when a current of 4A in the solenoid is switched of, the deflection on the B.G is 12 divisions. Calculate the sensitivity of B.G in divC^{-1}

3. A secondary coil of 500 turns is wrapped closely round the middle of a long solenoid which has 2000 turns per metre and area of cross section 9cm^2. the resistance of the secondary coil is 75Ω and is connected to a B.G of resistance 125Ω. When a current of 2.5A in the solenoid is reversed, a deflection corresponding to 28.3μC is registered. Calculate the permeability of air.

Transformer

Structure

It consists of an iron ring around which primary and secondary coils are wound. Ideally the primary coil has zero resistance and the secondary coil has high resistance.



Suppose an alternating voltage is applied to the primary coil and let V_p be the voltage across the primary at some instant. The magnetic flux linked with the circuit at any time t is BAN_s, where B is the instantaneous value of the magnetic flux density in the ring and A is the area of cross section in the ring.

The magnetic flux density B, is changing so that an emf is induced in the secondary and is given by $V_s = -\frac{d}{dt}(BAN_s) = -N_s A \frac{dB}{dt}$

$$V_s = -N_s A \frac{dB}{dt} \text{.......... ....}(i)$$

The primary coil has zero resistance and therefore the existence of current in the primary coil means that an emf, equal and opposite V_p is induced in the primary by self induction. There is no flux leakage so that the rate of change of flux in the primary is the same as that in the secondary. Therefore, due to self induction, an emf called back emf, ε_b is induced in the primary coil but $\varepsilon_b = V_p$ (since the resistance in the primary coil is zero)

$$\varepsilon_b = -\frac{d}{dt}(BAN_p) = -N_p A \frac{dB}{dt}$$

hence, $V_p = -N_p A \frac{dB}{dt} \text{.......... ......}(ii)$

Hence from (i) and (ii) $\frac{V_s}{V_p} = \frac{N_s}{N_p} = t$ = turn ratio

(1) If $N_s>N_p$, $V_s>V_p$ and the transformer is called step-up.

(2) If $N_s<N_p$, $V_s<V_p$ and the transformer is called step-down.

For an ideal transformer, the power in the primary is equal to the power developed in the secondary i.e. $V_pI_p = V_sI_s$, where I_p and I_s are flowing in the primary and secondary coils respectively.

Hence $\frac{V_s}{V_p} = \frac{I_s}{I_p}$

For a step up transformer, $I_s < I_p$

For a step down transformer, $I_s > I_p$

Note: Transformers operate only on a.c and not dc because dc does not produce changing magnetic flux and therefore no emf is induced in the secondary. In practice, transformers are not 100% efficient because of the energy losses.

Hence efficiency $= \frac{\sec ondarypowe \quad r}{primarypow \quad er} \times 100 \quad \%$

Efficiency = $\frac{I_s V_s}{I_p V_p} \times 100 \ \%$

Power losses/ energy losses in a transformer

1. Some of the energy is dissipated as heat due to the resistance of the coil (joule-ohmic energy loss)

This loss is minimized by using thick copper wires of low resistance.

2. Eddy currents circulating in the soft iron core produce heating and therefore reduces the amount of electrical power transferred to the secondary.

This is minimized by using a laminated core made of thin strips or laminars separated from each other by a layer of insulating varnish.

3. Hysteresis. The core or frame of iron ring is constantly being magnetized and demagnetized. Each time the direction of magnetization of the frame is reversed and some energy is wasted in overcoming internal friction.

4. some loss of energy occurs because a small amount of flux associated with the primary coil may not pass through the secondary coil. Leakage of magnetic flux.

This can be minimized by winding one coil on top of the other.

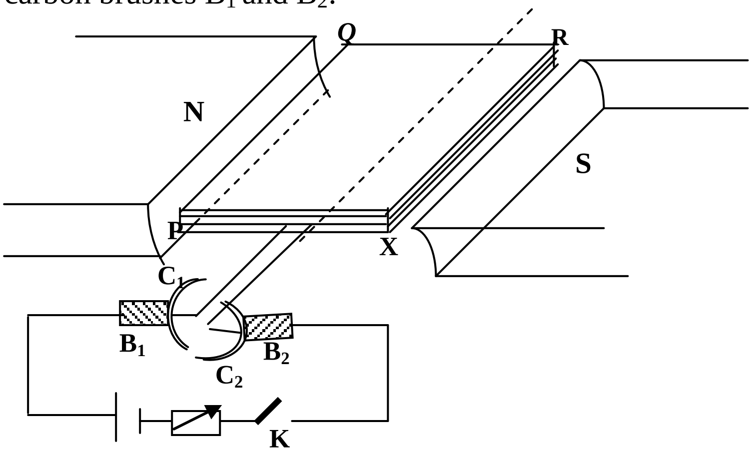
Electric power transmission

Electricity has to be transmitted over long distances from generating power stations to the consumers. This is causes some power loss in the transmission lines. This loss can be minimized if the power is transmitted at high voltage/ low current.

Electric power is steped up before transmission and stepped down at the consumers end.

Simple dc Motor

Structure : A simple dc motor consists of a rectangular coil of wire whose ends are connected to two halves C_1 and C_2 of a split ring commutator. The coil is mounted in a uniform magnetic field provided by a strong permanent magnet. The commutator halves C_1 and C_2 press against carbon brushes B_1 and B_2.



Action

Suppose the coil is horizontal at the instant switch K is closed. Current flows in the direction as shown. By Fleming's left hand rule, the left side of the coil experiences a downward force while side RX experiences an upward force. These two forces constitute a couple which causes the coil to rotate anticlockwise. When the coil reaches the vertical, the brushes B_1 and B_2 are pressing against the insulator or gap between C_1 and C_2, and current is cut off. The momentum of the coil however carries the coil beyond the vertical and the commutator halves reverse connections to the brushes. Current in the coil reverses. The forces on the left end of the coil is upward and that on right is downward. The coil continues rotating in the same direction.

Back emf in a motor

When the armature coil in a motor rotates, it cuts the magnetic flux of the magnet and an emf, called back emf, ε_b is induced in it which by Lenz's law opposes the applied p.d, V causing current I in the coil. If *r* is the coil resistance, then $V - \varepsilon_b = Ir$

Hence $VI - \varepsilon_b I = I^2 r$

Where VI is power supplied to the motor, I^2r is power dissipated as heat in the armature coil, $\varepsilon_b I$ is mechanical power output power of the motor or the rate of working against the induced emf.

The efficiency of the motor = $\frac{Mechanical\ \ power}{\sup\ \ pliedpower} \times 100\ \% = \frac{\varepsilon_b I}{VI} \times 100\ \% = \frac{\varepsilon_b}{V} \times 100\ \%$

Example

1. A dc motor has an armature resistance of 0.5Ω and is connected to a 240V supply. The armature current taken by the coil is 30A. calculate:

(i) the back emf generated by the motor (ii) the power supplied to the armature

(iii) the mechanical power developed by the motor. (iv) the efficiency of the motor

(v) State the power losses in a transformer and state how they are minimized.

Solution

(i) $V - \varepsilon_b = Ir$

$240 - \varepsilon_b = 30 \times 0.5$

$\varepsilon_b = 225V$

(ii) $P = VI = 240 \times 30 = 7200\ W$

(iii) mechanical power $= \varepsilon_b I = 225 \times 30 = 6750\ W$

(iv) Efficiency = $\frac{\varepsilon_b}{V} \times 100\ \% = \frac{225}{240} \times 100 = 93.8\%$

(v) energy losses in a dc motor

- energy loss due to the resistance of the coil. This minimized using thick Copper wires.
- Energy losses due to eddy currents. Minimized by using laminated core.
- Hysteresis loss in the core, reduced by using soft iron core .

2. The coil of a dc motor is mounted in a radial magnetic field of flux density 1T. the coil has 20 turns each of area 40cm^2 and total resistance 2Ω. Calculate the maximum angular velocity of the motor when working on a 240V supply and drawing current of 1A.

Solution

$V - \varepsilon_b = Ir$

$240 - \varepsilon_b = 1 \times 2$

$\varepsilon_b = 238V$

$\varepsilon_b = NA\omega B \sin \omega t$

$\varepsilon_b = NA\omega B$

$238 = 20 \times 40 \times 10^{-4} \times 1 \times \omega$

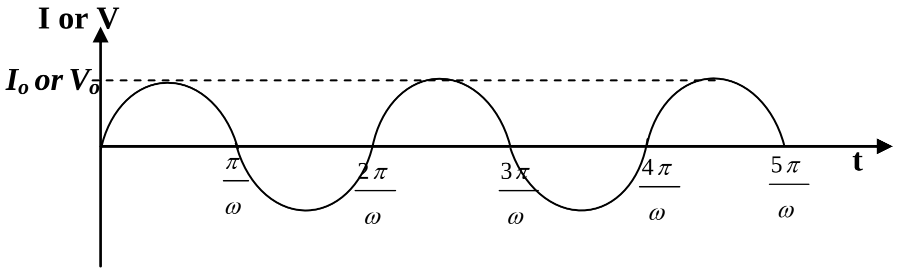
$\omega = 2975\ rads^{-1}$

ALTERNATING CURRENT CIRCUITS

Sinusoidal alternating currents and voltages

These are currents or voltages of the form $I = I_o \sin \omega t$ or $V = V_o \sin \omega t$, where I_o and V_o are the amplitudes or peak values. Alternating currents or voltages vary periodically with time in magnitude and direction.

A graph of V against t has the form:

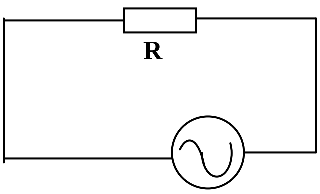


Root mean square value of alternating current or voltage (I_{rms} or V_{rms})

The root mean square value of alternating current is the value of steady current which dissipates electrical energy in a resistor at the same rate as the alternating current.
The root mean square value of alternating voltage is the value of steady voltage which dissipates electrical energy in a resistor at the same rate as the alternating voltage.

Relationship between peak current and I_{rms}

Consider a resistor in series with a.c source of electrical energy.



The instantaneous current I is given by $I = \frac{V_o}{R} \sin \omega t = I_o \sin \omega t$

Where $I_o = \frac{V_o}{R}$

The instantaneous power dissipated in the resistor, $P = I^2 R = (I_0 \sin \omega t)^2 R = I_o^2 R \sin^2 \omega t$

The average power over one cycle, $\langle P \rangle_T = \langle I_o^2 R \sin^2 \omega t \rangle_T = \frac{I_o^2 R}{2}$

Suppose I_{rms} is the value of steady current which dissipates electrical energy in the resistor at the same rate as a.c, the power dissipated by the steady current $= I_{rms}^2 R$

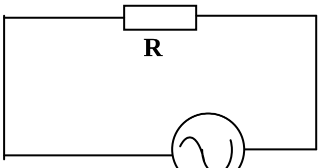
Hence $I_{rms}^2 R = \frac{I_o^2 R}{2}$

$\therefore I_{rms} = \frac{I_o}{\sqrt{2}}$

similarly, $V_{rms} = \frac{V_o}{\sqrt{2}}$

Resistor in a.c circuits

Consider a resistor in series with a.c source of electrical energy.



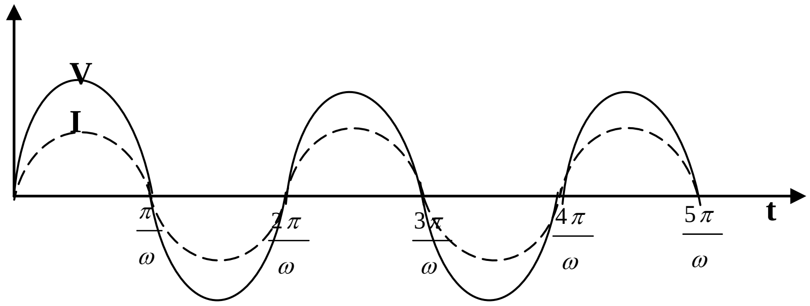
$V = V_o \sin \omega t$

The instantaneous current I is given by $I = \frac{V_o}{R} \sin \omega t = I_o \sin \omega t$

Where $I_o = \frac{V_o}{R}$

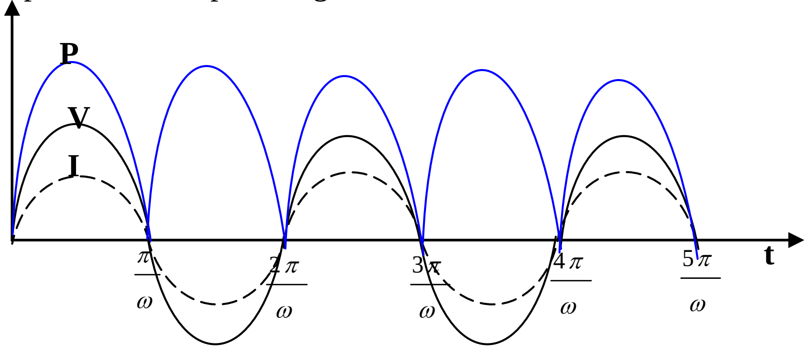
Hence V and I are in phase .

A graph of I and V against time , t is in the form



The instantaneous power dissipated in the resistor, $P = I^2 R = (I_0 \sin \omega t)^2 R = I_o^2 R \sin^2 \omega t$

Graph of I, V and power against time

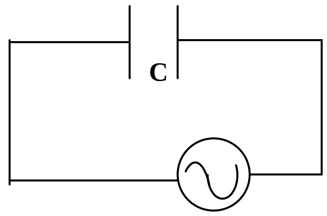


The average power over one cycle, $\langle P \rangle_T = \langle I_o^2 R \sin^2 \omega t \rangle_T = \frac{I_o^2 R}{2}$

Note: S*tudents should use calculus to prove the above expression.*

Capacitors in a.c circuits

Consider a capacitor of capacitance C, connected to an a.c source.



$V = V_o \sin \omega t$

Assuming that voltage is sinusoidal, $V = V_o \sin \omega t$

Hence charge Q on the plates of the capacitor is $Q = CV = CV_o \sin \omega t$

The current at any time t is equal to the rate of change of charge.

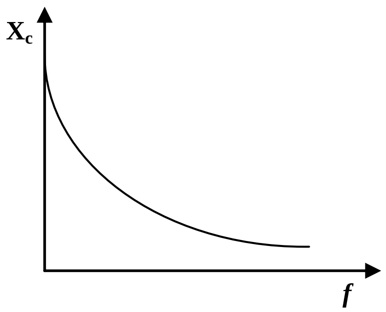
$$I = \frac{dQ}{dt} = \frac{d}{dt}(CV_o \sin \omega t) = CV_o \omega \cos \omega t = I_o \cos \omega t$$

Where $I_o = CV_o \omega$ = peak current.

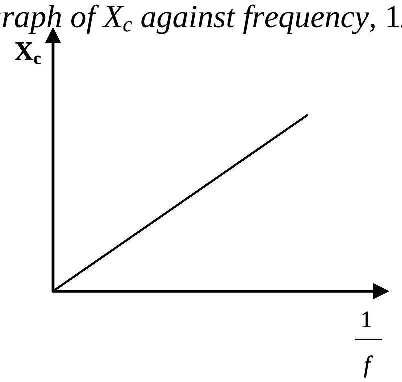
Capacitive reactance, $X_c = \frac{V_o}{I_0} = \frac{1}{C\omega} = \frac{1}{2\pi f C}$, f is the frequency of a.c.

Capacitive reactance is the opposition to flow of a.c in a capacitor.

A graph of X_c against frequency, f



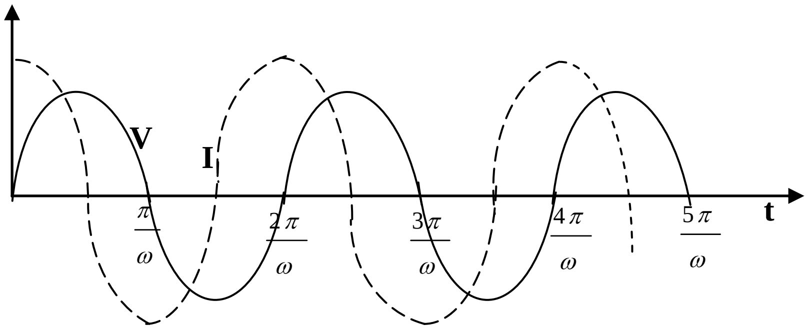
A graph of X_c against frequency, $1/f$



$V = V_o \sin \omega t$

$$I = I_o \cos \omega t = I_o \sin\left(\omega t + \frac{\pi}{2}\right)$$

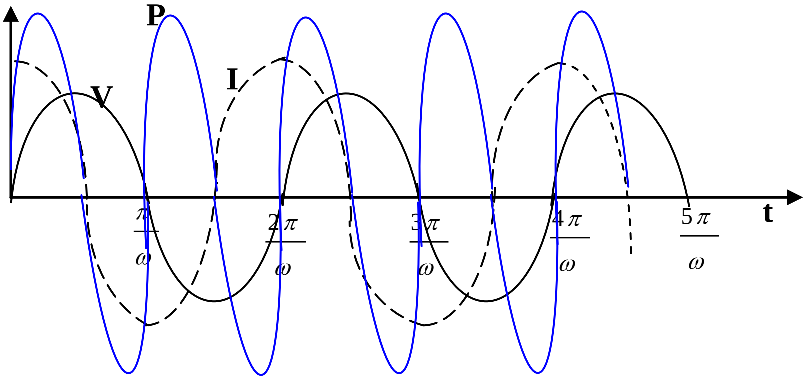
Hence current has the same frequency as applied voltage but leads the applied Voltage by $\frac{\pi}{2}$.



Instantaneous power developed in the capacitor,

$$P = VI = V_o \sin \omega t \times I_o \cos \omega t = V_o I_o \sin \omega t \cos \omega t = \frac{1}{2} V_o I_o \sin 2\omega t$$

Frequency of power is double the frequency of voltage.



Average power over one cycle $\langle P \rangle_T = \left\langle \frac{I_0 V_o}{2} \sin \omega t \right\rangle_T = 0$

Average Power over one cycle is zero. Hence a capacitor is a wattles component in a.c circuits.

Reason why a capacitor is a wattless component

During the first quarter cycle, the capacitor charges and energy is drawn from the source and stored in the electric field of the capacitor. During the second quarter cycle, the capacitor discharges and energy is returned to the source. During the third cycle, the capacitor charges in the opposite direction, again energy is stored in the electric field in the capacitor and in the last quarter, the capacitor discharges and energy returns to the source. Therefore in one cycle, there is no net energy stored in the capacitor.

Question: Explain why a capacitor allows the flow of a.c but not d.c.

Inductor in a.c circuits

A pure inductor is a coil of negligible resistance.

Assuming sinusoidal current I is passed through the inductor.

$$I = I_o \sin \omega t$$

the back emf, ε_b , due to the changing current is given by

$$\varepsilon_b = -L \frac{dI}{dt} = -L \frac{d(I_o \sin \omega t)}{dt} = -L \omega I_o \cos \omega t$$

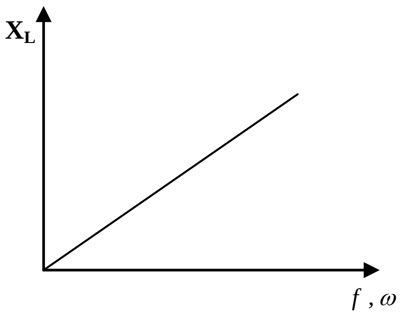
Assuming the inductor has zero resistance, then for current to flow, the applied p.d,V must be equal and opposite to the back emf, hence V = - ε_b

$$\therefore V = L \omega I_o \cos \omega t$$

or $\therefore V = V_o \cos \omega t$, Where $V_o = LI_o\omega$

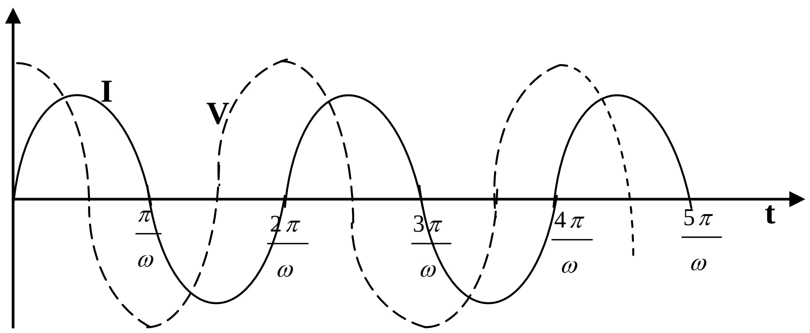
Reactance $X_L = \frac{V_o}{I_o} = L\omega$

Therefore in a capacitor, X_L has a linear relationship with ω



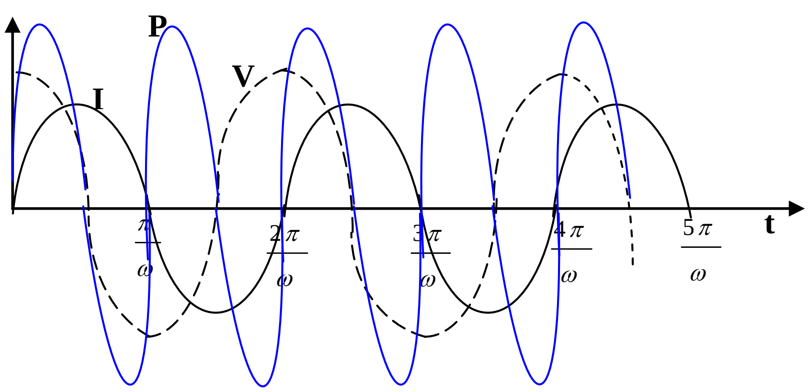
From the equations $I = I_o \sin \omega t$, $V = V_o \cos \omega t = V_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

Hence voltage has the same frequency as applied current but leads the applied current by $\frac{\pi}{2}$.



Power in inductive circuits

$$P = VI = V_o \sin \omega t \times I_o \cos \omega t = V_o I_o \sin \omega t \cos \omega t = \frac{1}{2} V_o I_o \sin 2\omega t$$



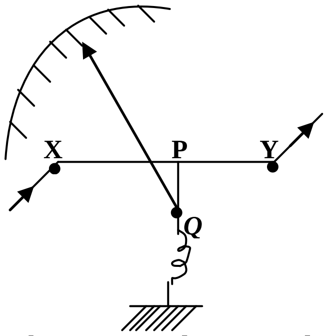
Average power over one cycle $\langle P \rangle_T = \left\langle \frac{I_0 V_o}{2} \sin \omega t \right\rangle_T = 0$

Why average power is zero

When the current rises, the back emf opposes the flow of current. The current flows against the back emf and therefore does work against it. The total work done in bringing the current ti its final value is stored in the magnetic field of the coil. It is liberated when the current collapses, therefore then the back emf tens to maintain the current and do external work. Hence, during the first quarter cycle, the magnetic field linking the coil is building up to the maximum value. Energy is supplied by the source and stored by the magnetic filed of the coil. During the second quarter cycle, the current in the coil decreases to zero. The energy that was stored in the coil is released to the source. In the third quarter cycle, the source supplies current in the reverse direction. The energy supplied by the source is stored in the magnetic field of the coil and in the 4^{th} cycle an equal amount of energy is restored to the store. Hence the energy dissipated over one cycle in a circuit is zero.

Instruments used to measure a.c

Hot wire ammeter

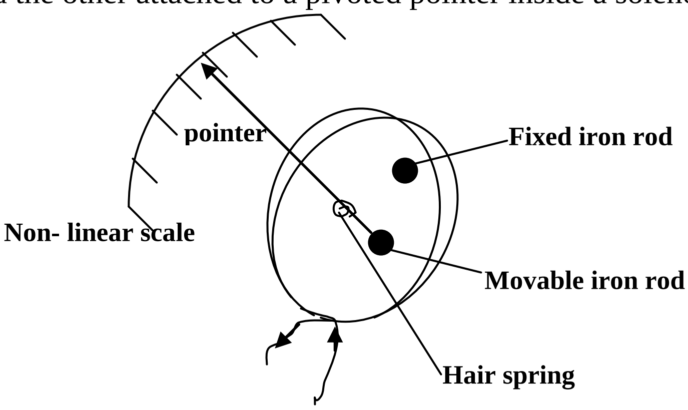


Current to be measured is passed through the resistance wire XY which heats up. The rise in temperature of the wire makes it expand and sag; the sag is taken up by a second fine wire P*Q*, which is held taut by a spring. He wire P*Q* passes round a pulley attached to the pointer of the instrument, which rotates as the wire XY sags.

The deflection of the pointer is roughly proportional to the average rate at which heat ids developed in the wire XY; it is therefore proportional to the average value of the square of the alternating current.

Moving Iron Ammeter (Repulsive type)

The repulsive type of a moving iron ammeter consists of two rods of soft iron, one fixed rigidly in position and the other attached to a pivoted pointer inside a solenoid carrying current to be measured.



The current I to be measured is passed through solenoid. Whatever the direction of the current, the iron rods are magnetized in the same sense and always repel each other. The repulsive forces cause the movable iron rod and hence the pointer to move until it is stopped by the restoring couple due to the hair springs.

For good approximation, the magnetization of iron rods is proportional to the current I. The force between the rods is proportional to I^2. hence the deflection of the pointer is proportional to the mean value of I^2. The scale reads root mean square values of current I and hence its non-linear scale.

Advantages of moving iron ammeter

- they are cheap
- can be used to measure both a.c and d.c.

Disadvantages of moving iron ammeter

- they have non-linear scale.

Examples

1. A capacitor of C, capacitance 1F is used in a radio circuit where frequency is 1000Hz and current is 2mA. Calculate the voltage across C.

Reactance $X_c = \frac{V_o}{I_0} = \frac{1}{C\omega} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 1000 \times 1 \times 10^{-6}} = 159\ \Omega$

$\therefore V = IX_c = 2 \times 10^{-3} \times 159 = 0.32\,V$

2. An inductor of 2H and negligible resistance is connected to a 12V mains supply, frequency 50Hz. Find the current flowing.

$X_L = L\omega = 2\pi fL = 2\pi \times 50 \times 2 = 628\ \Omega$

$\therefore I = \frac{V}{X_L} = \frac{12}{628} = 0.019\ A$

3. A sinusoidal a.c $I = 4\sin 100\pi t$ Amperes flows through a resistor of resistance 2Ω. Find the mean power dissipated in a resistor and hence deduce the rms value of current.

Average power = $\frac{I_o^2 R}{2}$, but I_o=4A

Hence average power = $\frac{4^2 \times 2}{2} = 16\ W$

Power dissipated by d.c $I_{rms}^2 R = 2I_{rms}^2$

Hence $2I_{rms}^2 = 16$

$I_{rms}^2 = 8$

$I_{rms} = 2\sqrt{2}\,A$

Questions

1. What is the rms value of alternating current which must pass through a resistor immersed in oil in a calorimeter so that the initial rate of rise of temperature of the oil is three times that produced when a direct current of 2A passes through the resistor under the same conditions?[3.46A]

2. A pure inductor of self inductance 1H is connected across an alternating voltage of 115V and frequency 60Hz. Calculate the

(i) inductive reactance (ii) inductive current (iii) peak current (iv) average power consumed. [377.1Ω, 0.3A. 0.43A, zero]